Moments of wave scattering by a rough surface

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(Received 18 December 1989; accepted for publication 21 June 1990)

This paper gives the first two moments of a wave field scattered by grazing incidence on a moderately rough surface. The expressions are derived for normally distributed surfaces with arbitrary spectrum, and are valid at depths that are large compared with the surface height. It is demonstrated that the first moment has a weak dependence on the surface fluctuation spectrum. The first moment is compared with Monte Carlo simulations, and gives close agreement. It is also shown that for a given degree of surface roughness the first moment retains the flat-surface reflection property of being determined by the distance from an "image source," i.e., the sum of the depths of source and receiver.

PACS numbers: 43.30.Hw

INTRODUCTION

In the study of rough surface scattering a central problem is to find the statistics of the scattered wave field[1-3] and relate them to those of the surface. Few results are known, in particular, for the case of a wave Ψ multiply scattered by grazing incidence on a very rough one-dimensional surface; expressions for the moments have been given[3] for the perturbation regime of small surface heights, and in the near-surface region[4] for slightly greater roughness.

In this paper equations are derived, following the method proposed previously,[5] for the first and second moments of the scattered field due to a Gaussian beam. These expressions are valid for moderate rms surface height ϕ, at depths much greater than ϕ. The surface is assumed to be normally distributed, with arbitrary autocorrelation function. Wave propagation in this regime is well described by the parabolic equation method,[6] which expresses the field in terms of a Green's function. In deriving the equations here, we first obtain the correlation between the Green's function $G(\bar{r},\bar{r}')$ and the incident field $\Psi_{\rm inc}(\bar{r}'')$ where $\bar{r}'$, $\bar{r}''$ are points on the random surface. The moments of the scattered field are then expressed in terms of this correlation, using an approximate form[5] for the vertical derivative $\partial\Psi/\partial z$ along the surface. The equations obtained are based partly on numerical discretization, but express the dependence of the moments on the surface statistics explicitly. The equation for the first moment is compared with Monte Carlo simulations for a surface with Gaussian spectrum, and gives good agreement. The first moments are compared for surfaces with Gaussian and power-law spectra, and are found to depend only weakly on the surface spectrum.

For a source at a depth z_0 below a flat surface as is well known the scattered field Ψ_s at depth z_1 equals the field that would be due directly to an "image source;" thus Ψ_s is determined by $z_0 + z_1$. It is shown here that when the surface becomes rough although the mean field depends strongly on ϕ, it retains this property. This result is confirmed by simulations for much larger values of ϕ.

The parabolic equation method and other preliminaries are given in Sec. I, and in Sec. II the solutions are described and computational results given.

I. MATHEMATICAL PRELIMINARIES AND EQUATIONS

In this section we set out the details of the parabolic equation method for simulation of surface scattering,[6] following the notation used previously,[5] and recall the results which are needed.

A. Parabolic equation method and numerical solution

We consider the two-dimensional scattering problem, i.e., from a one-dimensional surface. We treat the case in which the field is incident at low grazing angles, and is governed by the parabolic wave equation. The coordinate system (x,z) is taken as usual, where x is horizontal, $x\geqslant 0$, and z is the vertical, decreasing downwards. Let $\bar{r} = (x,z)$. The source will be centered around $\bar{r}_0 = (0,0)$, and its mean distance from the surface will be denoted z_0. Let $h_1(x)$ be the rough surface, and let $h \equiv z_0 - h_1$, so that h has mean zero. The derivative of h is assumed to be bounded. The equations are derived here for normally distributed surfaces. Denote the rms of the random component $h(x)$ of the surface by ϕ, and its correlation length by L. Here, $\Psi_{\rm inc}(\bar{r})$ denotes the (complex) incident wave field. It will be assumed that $\Psi_{\rm inc} = 0$ at the surface for $x \leqslant 0$, and that $h_1(x)$ is a pressure-release surface. Here, $\Psi_s(\bar{r})$ denotes the wave field scattered from the surface and $\Psi(\bar{r})$ the total field, so that $\Psi = \Psi_s + \Psi_{\rm inc}$, and $\Psi = 0$ at the surface. The surface derivative $\partial\Psi/\partial z$ may be denoted by Ψ'. The governing equations for the parabolic equation method are

$$\Psi_{\rm inc}(\bar{r}) = -\int_0^x \left(G(\bar{r};\bar{r}')\, \frac{\partial\Psi}{\partial z'} \right)_{\bar{r},\bar{r}' \text{ at surface}} dx', \qquad (1)$$

which may be written in operator form as

$$\Psi_{\rm inc} = A\Psi'$$

and

$$\Psi_s(\bar{r}) = \int_0^x \left(G(\bar{r};\bar{r}')\, \frac{\partial\Psi}{\partial z'} \right)_{\bar{r}' \text{ at surface}} dx'. \qquad (2)$$

Here G is the Green's function given by

$$G(x,z;x',z') = \begin{cases} \frac{1}{2}\sqrt{\frac{i}{2\pi k(x-x')}}\exp\left(i\frac{k}{2}\frac{(z-z')^2}{x-x'}\right), & \text{for } x \geqslant x', \\ 0, & \text{for } x < x', \end{cases} \quad (3)$$

where $\vec{r} \equiv (x',z') \equiv (x',h_1(x'))$ along the surface and k is the wave number.

The incident wave is a simple Gaussian beamed source of width w traveling at a small angle θ to the surface:

$$\Psi_{\text{inc}}(x,z) = \frac{w}{\sqrt{w^2 + 2ix/k}} \times \exp\left(-\frac{2z^2 + ikSw^2(Sx - z)}{2(w^2 + 2ix/k)}\right), \quad (4)$$

where $S = \sin(\theta)$. In the computational examples that follow we have taken $k = 1$ and $w = 8$, and have considered the forward-traveling case $\theta = 0$. In most of the results the correlation length is $L = 8$, which is of the order of a wavelength.

We need some additional notation: Let $\Psi_0(x) = \Psi_{\text{inc}}(x,z_0)$ be the incident field along z_0, and let Ψ_0' and A_0 denote the (deterministic) forms of Ψ' and A, respectively, which would be due to a flat surface at z_0. For any of these quantities f, say, $\langle f\rangle$ will denote its ensemble average, taken over realizations of the surface. The normalized autocorrelation function of the surface (with mean removed) is given by

$$\rho(\xi) = \langle h(x)h(x+\xi)\rangle/\phi^2.$$

We will make use of the autocorrelation functions for surfaces with Gaussian and fourth-order power-law spectra. For the Gaussian case this is

$$\rho(\xi) = \exp(-\xi^2/L^2),$$

and the autocorrelation function corresponding to the power-law spectrum is

$$\rho(\xi) = (1 + |\xi|/L)\exp(-|\xi|/L).$$

The integrals in Eqs. (1) and (2) both contain weak (i.e., integrable) singularities. We give brief details here of the numerical treatment[5] of these equations, from which Monte Carlo simulations are run for comparison with the moment equations.

Equation (1) is discretized with respect to x, using N equally spaced points $x_1,\ldots,x_N$, say. The integral is divided into corresponding subintervals, on each of which product integration is applied,[7] treating the function Ψ' as constant. A may thus be represented by an $N \times N$ lower-triangular matrix with entries

$$a'_{i,j} = \int_{x_{j-1}}^{x_j} \frac{1}{2}\sqrt{\frac{i}{2\pi k(x_i - x')}} \times \exp\left(\frac{ik}{2}\frac{[h(x) - h(x')]^2}{x - x'}\right)dx',$$

for $i \geqslant j$, and this is inverted numerically.

Equation (2) for the scattered field is evaluated using a semi-analytical approach: the integral is again divided into subintervals $[x_r,x_{r+1}]$ and the argument is expanded in terms of $(x' - x_r)$. By making a change of variables and integrating by parts the integrand may be expressed in terms of Fresnel integrals.

B. Approximations

The results here make use of several approximations discussed previously.[5] The first is the following expression for the incident field at the surface:

$$\Psi_{\text{inc}}(x) \simeq \Psi_0(x)\exp\left(\frac{4z_0h(x) - ikSw^2h(x)}{2(w^2 + 2ix/k)}\right). \quad (5)$$

This neglects a factor $\exp[ikh^2/(w^2 + 2ix/k)]$. It holds for $\phi^2 \ll w^2$ provided the source is sufficiently far from the surface, i.e., $\phi \ll z_0$. (When ϕ is comparable with z_0, although the error term may remain small, it represents a significant part of the random variation in Ψ_{inc}, and neglecting it may then corrupt the statistics.)

We will be concerned with the single random variable form $G(x,z,x',h_1(x'))$ of the Green's function as it appears in Eq. (2). Let $z' = z_0 - z$ so that z' is the depth (relative to the mean surface level). When z' is large compared with ϕ, the exponent in the Green's function can be approximated as follows:

$$\frac{ik(z' - h(x'))^2}{2(x - x')} \simeq ik\frac{z'^2 - 2z'h(x')}{2(x - x')}. \quad (6)$$

This again holds for any $k\phi$ provided z' is sufficiently large. (The error is a factor $\exp[ikh^2/2(x - x')]$; although this exponent becomes large as $x' \to x$ it gives rise to negligible errors in Ψ_s as the phase variation in G is dominated by approximate exponent (6).)

The third approximation replaces the operator A in (1) by its deterministic form A_0:

$$\Psi_{\text{inc}} \cong A_0\Psi'. \quad (7)$$

It follows from this that the mean of Ψ' may be approximated by $\langle\Psi'\rangle \cong A_0^{-1}\langle\Psi_{\text{inc}}\rangle$, which was shown[5] in this scattering regime to be fairly accurate for $k\phi$ up to around 2. The expression (7) is a crucial step, since it enables us to write the integrand in Eq. (2) as an explicit function of surface heights $h(x)$.

As described above the operator A_0 may be represented as a matrix and inverted numerically, to give A_0^{-1}. A_0 is a Toeplitz matrix, i.e., one which is lower triangular and constant along the diagonal and each subdiagonal, with entries

$$a'_{i,j} = \int_{x_{j-1}}^{x_j} \frac{1}{2}\sqrt{\frac{i}{2\pi k(x_i - x')}}dx' = \sqrt{i/2\pi k}\left(\sqrt{x_i - x_{j-1}} - \sqrt{x_i - x_j}\right).$$

The inverse A_0^{-1} is therefore also a Toeplitz matrix, and its entries may be written $a_{i,j} = a_{i-j}$, where the a's are deterministic constants. Thus for any x_n we have

$$\Psi'(X_n) \cong \sum_{i=1}^{n} a_{n-i} \Psi_{\rm inc}(x_i), \tag{8}$$

where $X_n = (x_n + x_{n-1})/2$. This discrete representation of A_0^{-1} is important for us because it allows the correlation $\langle G\Psi' \rangle$ to be expressed as a sum of terms $\langle G\Psi_{\rm inc} \rangle$. [Note that the equation $\Psi_{\rm inc} = A_0 \Psi'$ has the solution[7]

$$\Psi'(x) = \frac{1}{2\pi} \sqrt{\frac{i}{2\pi k}} \frac{d}{dx} \left(\int_0^x \frac{\Psi_{\rm inc}(x')}{(x-x')^{1/2}} dx' \right).$$

Although this formula is analytical, any potential gain in accuracy over (8) is lost in discretization. However it is a more convenient form from which to derive explicit coefficients a_{n-i}. For example if the derivative is approximated using a backward finite difference and the integral discretized as before we can write $a_k = (1/\pi\delta)(a'_{k+1,1} - a'_{k,1})$ for $k > 0$ and $a_0 = a'_{1,1}/\pi\delta$, where $a'_{j,1}$ is as above and δ is the constant step size $(x_j - x_{j-1})$.]

II. MOMENTS OF THE SCATTERED FIELD

We can now derive the equations for the first and second moments of Ψ_s. We first give expressions for the moments of the product of the Green's function G with the incident field $\Psi_{\rm inc}$ at the surface.

A. Statistics of $G\,\Psi_{\rm inc}$

Consider again the form $G(x,z,x',h_1(x'))$ of the Green's function appearing in Eq. (2), which we may write $G(x,z,x')$. Denote by F the correlation function

$$F(x,x',x'',z) = \langle G(x,z,x') \Psi_{\rm inc}(x'') \rangle.$$

Similarly denote by F_2 the four-point correlation

$$F_2(\bar{x};\bar{y}) = \langle G(x,z_1,x') G^*(y,z_2,y') \times \Psi_{\rm inc}(x'') \Psi^*_{\rm inc}(y'') \rangle,$$

where $\bar{x} = (x,x',x'',z_1)$, $\bar{y} = (y,y',y'',z_2)$, the asterisk denotes complex conjugation, and all the coordinates except for z_1, z_2 are x coordinates.

Consider first the function $F(x,x',x'',z)$, and again denote the depth by $z' = z_0 - z$. Using Eqs. (5) and (6), we can write $G(x,z,x')\Psi_{\rm inc}(x'')$ as

$$G(x,z,x')\Psi_{\rm inc}(x'') \cong c(x,x',x'',z') \exp[b(x'')h(x'') - a(x,x',z')h(x')], \tag{9}$$

where

$$a(x,x',z') = ikz'/(x-x'),$$

$$b(x'') = (4z_0 - ikSw^2)/2(w^2 + 2ix''/k),$$

and

$$c(x,x',x'',z') = \frac{1}{2} \sqrt{\frac{i}{2\pi k(x-x')}} \exp\left(\frac{ikz'^2}{2(x-x')} \right) \Psi_0(x'').$$

Now the exponent in (9) is itself a normal random variable X, whose variance $\langle X^2 \rangle$ is given by

$$\sigma^2(x,x',x'',z_0,z') = \phi^2[a^2 + b^2 - 2ab\rho(x'-x'')].$$

The mean of the exponential in (9) is just $\exp(\sigma^2/2)$, (e.g., Papoulis[8]), and therefore, since c is deterministic,

$$F(x,x',x'',z) \cong c(x,x',x'',z') \exp(\sigma^2/2). \tag{10}$$

The function F_2 may be calculated similarly. Writing $a(x,x',z'_1)$ as $a(\bar{x})$, $b(x'')$ as $b(\bar{x})$, and so on, we have

$$F_2(\bar{x};\bar{y}) = c(\bar{x}) c^*(\bar{y}) \exp(\sigma_2^2/2), \tag{11}$$

where σ_2^2 is the variance of

$$b(\bar{x})h(x'') - a(\bar{x})h(x') + b^*(\bar{y})h(y'') - a^*(\bar{y})h(y'),$$

and is given by

$$\begin{aligned}\sigma_2^2 = \phi^2\{&a^2(\bar{x}) + b^2(\bar{x}) + a^2(\bar{y}) + b^{*2}(\bar{y}) \\ &- 2a(\bar{x})[b(\bar{x})\rho(x'-x'') + a(\bar{y})\rho(x'-y') \\ &+ b^*(\bar{y})\rho(x'-y'')] + 2b(\bar{x})[a(\bar{y})\rho(x''-y') \\ &+ b^*(\bar{y})\rho(x''-y'')] + 2a(\bar{y})b^*(\bar{y})\rho(y'-y'')\}.\end{aligned}$$

It is easy, but notationally cumbersome, to extend this to the higher moments of $G\,\Psi_{\rm inc}$, and this will not be done here.

B. First moment of Ψ_s

Consider the integral (2) for a fixed $x = x_n$. We can divide the region of integration into subintegrals of the form

$$\int_{x_{r-1}}^{x_r} G(x_n,z,x')\Psi'(x')dx'.$$

Since Ψ' is smoothly varying each subintegral can be written approximately as

$$\Psi'(X_r) \int_{x_{r-1}}^{x_r} G(x_n,z,x')dx',$$

where $X_r = (x_r + x_{r-1})/2$. Applying Eq. (8) and rearranging this becomes

$$\sum_{j=1}^{r} a_{r-j} \int_{x_{r-1}}^{x_r} G(x_n,z,x')\Psi_{\rm inc}(x_j)dx'.$$

The mean of this is obtained by taking the average of the integrand, and thus, averaging both sides of Eq. (2), we get

$$\langle \Psi_s(x_n,z) \rangle \cong \sum_{r=1}^{n} \sum_{j=1}^{r} \int_{x_{r-1}}^{x_r} a_{r-j} F(x_n,x',x_j,z)dx', \tag{12}$$

where F is given by (10)

C. Second moment of Ψ_s

The full second moment $\langle \Psi_s(x,z_1)\Psi_s^*(y,z_2) \rangle$ may be obtained similarly. Consider the product $\Psi_s(x_m,z_1)\Psi_s^*(x_n,z_2)$. Applying Eq. (2) to each term, dividing again into subintegrals, and multiplying we get

$$\Psi_s(x_m,z_1)\Psi_s^*(x_n,z_2) = \sum_{r=1}^{m} \sum_{s=1}^{n} \int_{x_{r-1}}^{x_r} \int_{x_{s-1}}^{x_s} G(x_m,z_1,x') G^*(x_n,z_2,x'') \times \Psi'(x')\Psi'^*(x'')dx'\,dx''.$$

Each of the double subintegrals can be written, using Eq. (8), as

$$\sum_{j=1}^{r}\sum_{k=1}^{s} a_{r-j}a_{s-k}\int_{x_{r-1}}^{x_r}\int_{x_{s-1}}^{x_s} G(x_m,z_1,x')$$
$$\times G^*(x_n,z_2,x'')\Psi_{\rm inc}(x_j)\Psi^*_{\rm inc}(x_k)dx'\,dx''.$$

On averaging, the integrand becomes $F_2(x_m,x',x_j,z_1;x_n,x'',x_k,z_2)$ and so the second moment of Ψ_s is approximately

$$\sum_{r=1}^{m}\sum_{s=1}^{n}\sum_{j=1}^{r}\sum_{k=1}^{s} a_{r-j}a_{s-k}$$
$$\times\int_{x_{r-1}}^{x_r}\int_{x_{s-1}}^{x_s} F_2(x_m,x',x_j,z_1;x_n,x'',x_k,z_2)dx'\,dx''.$$

Rearranging the order of summation and taking the coefficients under the integral signs this becomes

$$\sum_{j=1}^{m}\sum_{k=1}^{n}\sum_{r=j}^{m}\sum_{s=k}^{n}\int_{x_{r-1}}^{x_r}\int_{x_{s-1}}^{x_s} a_{r-j}a_{s-k}$$
$$\times F_2(x_m,x',x_j,z_1;x_n,x'',x_k,z_2)dx'\,dx''.$$

Each coefficient a_t may be regarded as the value at x_t of a step-function $\alpha(x)$, so that $\alpha(x)\equiv a_t$ for $x_t \leqslant x < x_{t+1}$. This allows us to replace the double integral and the two inner summations by a double integral over a larger region, and we get

$$\sum_{j=1}^{m}\sum_{k=1}^{n}\int_{x_{j-1}}^{x_m}\int_{x_{k-1}}^{x_n}\alpha(x'-x_{j-1})\alpha(x''-x_{k-1})$$
$$\times F_2(x_m,x',x_j,z_1;x_n,x'',x_k,z_2)dx'\,dx''. \quad (13)$$

Although numerical evaluation of Eq. (13) is feasible it is computationally intensive, and further analytical treatment of the integral is clearly required.

D. Computational results

The expression (12) for the first moment was evaluated and compared with simulations, taking the average over many realizations, for various depths and degrees of surface roughness. An example of this comparison is shown in Fig. 1 as a function of x for $k\phi = 1.6$. The deterministic (flat surface) form is also given. Note that the surface roughness causes a slight shift in the position of the peak of the averaged field $\langle\Psi_s\rangle$. In Fig. 2 simulations are compared with equation 12 for $k\phi = 1.5$ as a function of depth z', to illustrate the increasing accuracy of the approximation with z'. [The simulations were carried out as described in Sec. I above. The flat surface curves are given by the exact solution $\Psi_s(z) = -\Psi_{\rm inc}(2z_0 - z)$.]

In order to examine the behavior of the mean with changes in the surface statistics, Eq. (12) was compared for two different correlation functions. Figure 3 gives the amplitude of $\langle\Psi_s\rangle$ as a function of x for a Gaussian and a power-law surface, with $k\phi = 1.6$, and the curves show only a slight difference. The corresponding real parts are compared in Fig. 4.

We now consider briefly the effect of changing the depths of source and receiver so that their sum is kept fixed. As mentioned above this sum would represent distance from an image source in the case of a flat surface. In Fig. 5 the real part of $\langle\Psi_s\rangle$ is shown, calculated from (12), for three cases with $z_0 + z' = 42.4$ and $\phi = 1.4$. The resulting curves are indistinguishable. A similar comparison was carried out using simulations, for $\phi = 5$ and $z_0 + z' = 67.4$, taking the average over 400 realizations. The real parts are shown in Fig. 6, and show the same behavior. The flat surface curves in each case are given for comparison.

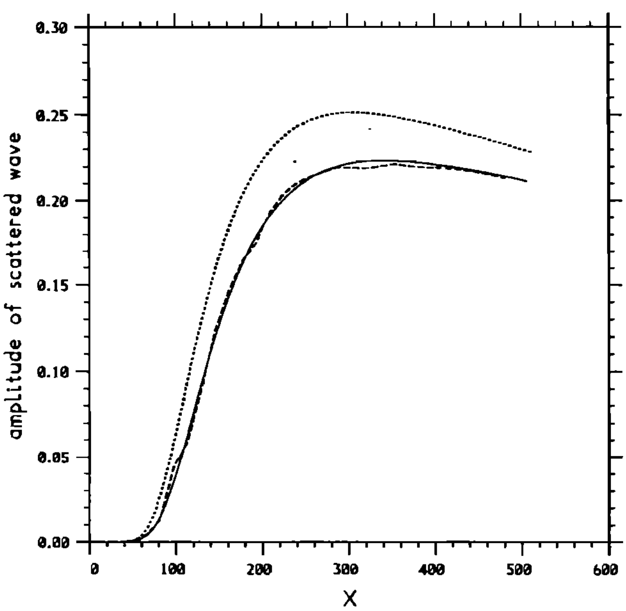

FIG. 1. Comparison between predicted form (full line) of the amplitude of $\langle\Psi_s(x,z)\rangle$ as a function of x and the value from simulations (dashed line) for $k\phi = 1.6$ at a depth of $z' = 16.0$, using 200 realizations. The form of $|\Psi_s|$ due to a flat surface is also shown (dotted line).

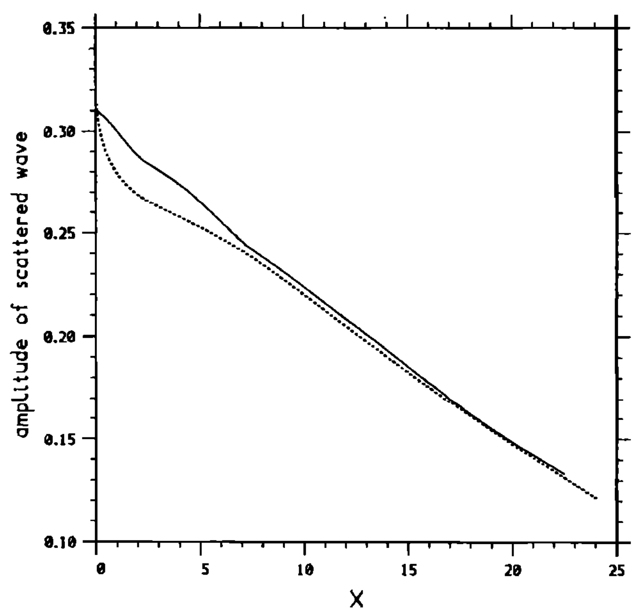

FIG. 2. Predicted amplitude of $\langle\Psi_s(x,z)\rangle$ (full line) compared with simulations (dashed line) as a function of depth z', with $k\phi = 1.5$.

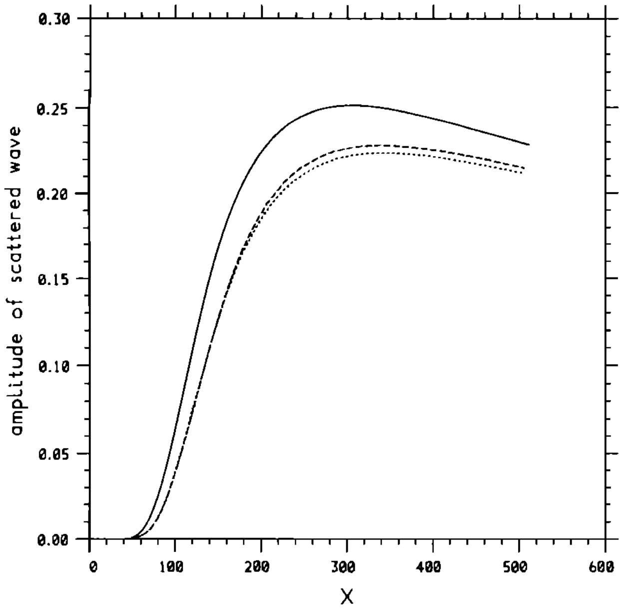

FIG. 3. Amplitude of the mean as a function of x for surfaces with Gaussian (dotted line) and fourth-order power law (dashed line) spectra, with $k\phi = 1.6$ at $z' = 16.0$, together with the flat surface form of $|\Psi_s|$ (full line).

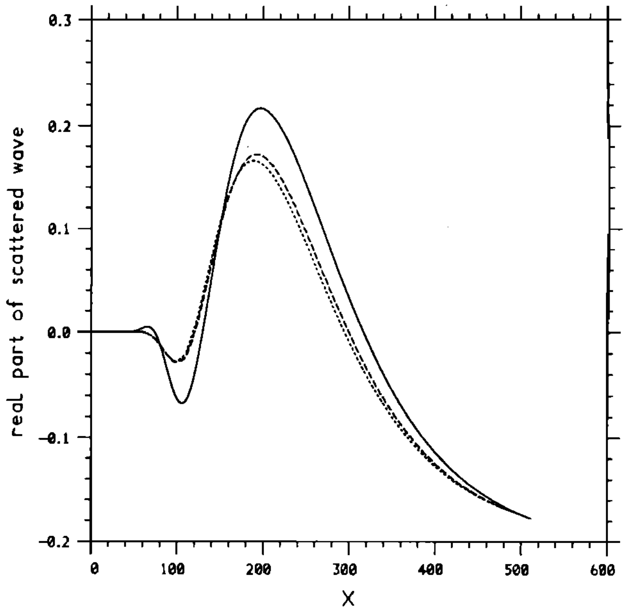

FIG. 4. Real part of the mean as in Fig. 3, for Gaussian (dotted line) and fourth-order power law (dashed line) spectra, together with the flat surface form (full line).

In the above examples the integral in Eq. (12) was evaluated numerically. As with the stochastic analog [Eq. (2)] there is a weak singularity at $x' = x_n$, where the phase changes with $(x_n - x')^{-1}$. For $\phi \neq 0$, however, the amplitude of the integrand approaches zero exponentially as $x' \to x_n$. We can estimate the contribution of the integral over a small interval $[x_n - \delta, x_n]$ as follows: use product-integration to take the slowly varying factors outside the integral, and replace $x_n - x'$ by the variable $y = (x_n - x')^{-1}$. The integral then takes the form

$$\int_{1/\delta}^{\infty} y^{-3/2} \exp(\alpha y - dy^2) dy,$$

where d is real and positive and α is complex. This is bounded by

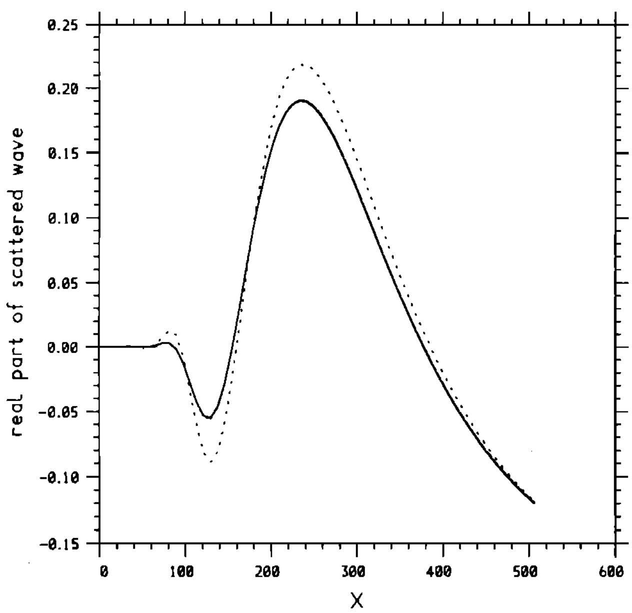

FIG. 5. Real part of the mean as a function of x for a surface with Gaussian spectrum for $\phi = 1.4$, for three pairs of depths (z_0, z'), with $z' = 10$ (full line), 20 (dotted line), and 25 (dashed line), and in each case $z_0 = 42.4 - z'$. The flat surface form of $|\Psi_s|$ is also shown (widely spaced dotted line).

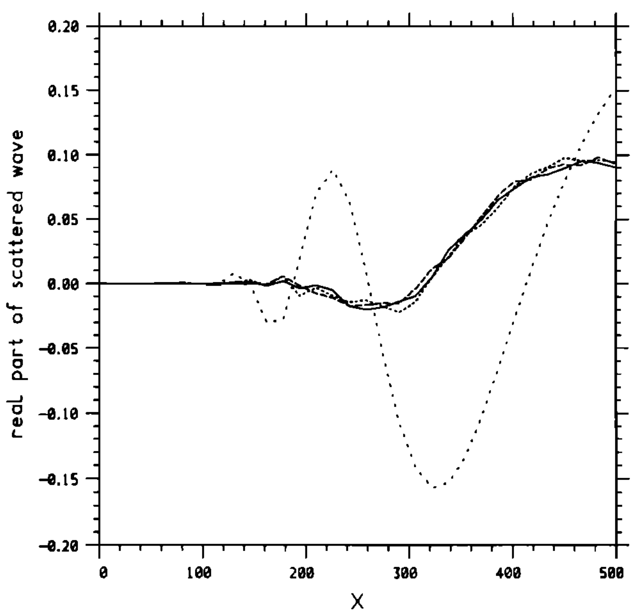

FIG. 6. Real part of the mean as a function of x for a surface with Gaussian spectrum for $\phi = 5$, for three pairs of depths (z_0, z'), with $z' = 20$ (full line), 30 (dotted line), and 35 (dashed line), and in each case $z_0 = 67.4 - z'$. The flat surface form is also given (widely spaced dotted line).

$$\delta^{3/2}\int_{1/\delta}^{\infty}\exp(\alpha y - dy^2)dy,$$

which may be expressed in terms of Fresnel integrals [e.g., Gradshteyn and Ryzhik[9]] and decreases rapidly with δ.

III. SUMMARY

Equations have been given for the first two moments of a wave scattered by grazing incidence on a rough surface, and the equation for the first moment agrees well with Monte Carlo simulations. The formulation of these equations is partly numerical, but they express the dependence of the moments on the surface statistics explicitly. It was found that for moderate surface heights changes in the surface fluctuation spectrum have relatively little effect on the first moment. It was seen that the scattering causes a shift in the position of the peak of the reflected wave, which is known from simulations to increase with surface height. It was also seen that for given surface roughness the mean scattered field is determined by the sum of the depths of source and receiver. Such features are of some importance, and the full spatial dependence of the mean and second moment must be addressed in future work.

Note that the range of validity of these expressions is slightly increased if the approximation (7) is replaced by $\Psi_{\rm inc} \cong \langle A \rangle \Psi'$, where $\langle A \rangle$ is known.[4] However the explicit dependence of the moments on the surface statistics then becomes obscured when $\langle A \rangle$ is inverted, without significantly increasing the rms surface heights for which these equations hold.

ACKNOWLEDGMENTS

This work was carried out with the financial support of the Natural Environment Research Council of the U.K. The author is grateful to Dr. Barry Uscinski and Professor Eric Jakeman for useful discussions, and to the referee for several very helpful suggestions and comments.

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