Paper

Lateral instability of long-span prestressed concrete beams on flexible bearings

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Synopsis

The paper describes the method of calculation of the buckling load of beams on flexible bearings due to the beam's self-weight. Methods are given that allow calculation of the bearing stiffness needed to prevent instability, as well as the additional curvature and additional stresses due to the growth of initial imperfections at loads less than the critical load. Methods of providing temporary support and of jacking beams into position are also discussed.

Notation

A	plan area of bearings
E	Young's modulus of concrete
E_R	Young's modulus of unconfined rubber
f_b, f_{b1}, f_{b2}	are parameters used in determining rotational stiffness of bearing
I_y	second moment of area about the beam section's minor axis
K	rotational stiffness of bearings
k	radius of gyration of plate in a laminated bearing
L	length of beam
M	moment about bearings
t	thickness of rubber layer in laminated bearing
u	total lateral displacement due to beam's weight
v	lateral deflection measured in the minor-axis direction (which rotates with θ)
v_{av}	average lateral deflection
v_0	initial lateral imperfection
w	self-weight of beam/unit length
w_{cr}	critical self-weight of beam to cause buckling/unit length
X	position of extreme fibre from minor axis
y_b	distance of bottom fibre of beam below centroid of beam
z	position along the beam
κ_{ms}	midspan curvature about minor axis
ϕ	rotation of bearings
$\Delta\sigma$	additional stress due to minor-axis buckling effects

Introduction

In recent years, precast concrete beams have been produced for bridges with significantly higher spans than in the past. In the UK, the longest standard beams available are designed for spans of 40m, but in Canada, for example, spans of up to 55m can be erected using standard sections, and larger beams can be designed if transport is not a problem. To minimise weight for transportation, these beams have only residual flanges, which leads to a low minor-axis stiffness. Such beams are susceptible to lateral–torsional buckling failures when simply supported[1,2] or a lateral toppling mechanism when hanging from crane cables[3]. There have, also, been problems on site[4,5] with beams toppling sideways when placed on (temporary) bearings with insufficient lateral restraint. These, and other unreported problems on site, led the authors to study the general problem of beam instability which is exacerbated by additional flexibility allowed by rubber bearings; this work addresses that problem.

The paper determines the relationship between the self-weight at which the beam would become unstable and the rotational stiffness of the support. It also shows how the effects of initial imperfections in the beam will produce minor-axis curvature, and hence additional stresses that could, in limiting cases, cause cracking of the beam, and hence a possibly catastrophic failure due to loss of stiffness.

The paper also draws together the information needed to determine the rotational stiffness of laminated rubber bearings (which are not normally quoted in bearing catalogues) and discusses methods for temporary supports and jacking of beams. Some worked examples are also given.

It is not expected that the theory given here will limit the design of the beams themselves (provided due account has been taken of the buckling criteria given elsewhere), but it may control the choice of bearing or temporary support condition.

Bridge bearings are normally designed to allow rotation in the bridge's working state. They must allow rotation about the bridge's horizontal bending axis (i.e. parallel to the face of the abutment) and must allow movement due to thermal effects. Bending about an axis normal to the face of the abutment (i.e. along the line of beam in the case of a square bridge) is not a design condition, since in the permanent condition, when multiple beams are joined by a top slab, no rotation about that axis can occur. However, in the temporary condition, things are different. Each beam sits on its own bearings at each end, and these bearings must provide rotational restraint about an axis normal to the abutment. In certain cases (as, for example, when the beam is being moved from temporary supports to permanent bearings), all the restraint may have to be provided by the bearing at one end only.

If the beam is supported on bearings that have some flexibility about this normal axis, the beam can rotate about the bearings. Part of the beam's weight is then acting about the beam's minor axis, which can then cause further sideways deflection. In the limit, this deflection can cause buckling.

Theory

Consider a beam supported at its ends on bearings that have rotated through an angle ϕ, as shown in Fig 1. The beam's own weight will act in part in the minor-axis direction, which will cause the beam to deflect in that direction. In accordance with the observations associated with the hanging beam problem[3], it will be assumed that the beam rotates as a rigid body with no variation in twist along the length of the beam, although minor-axis flexibility is permitted. This is valid for the relatively high torsional stiffness of concrete sections, but may not be valid for open steel sections, which tend to be much thinner. The analysis is thus lateral bucking theory associated with rotation.

Of particular concern here is the additional lateral deflection caused by the rotation ϕ due to the beam's self-weight w/unit length. It will be assumed that the bearing offers negligible resistance to rotation about a vertical axis, so, for the purposes of lateral deflection, it acts as a simple support. The lateral deflected shape of a simply supported beam under a uniformly distributed load $w\sin\phi$ is

$$v = \frac{w\sin\phi}{EI_y}\left[\frac{z^4}{24} - \frac{Lz^3}{12} + \frac{L^3z}{24}\right] \qquad ....(1)$$

where

- z is the distance from one end
- L is the span of the beam
- EI_y is the minor-axis flexural stiffness of the beam

This deflection can be integrated over the length of the beam to give the average displacement.

$$v_{av} = \frac{w\sin\phi L}{120EI_y} \qquad ....(2)$$

The lateral displacement (u) of the resultant of the beam's weight is made up of a component due to the rotation about the base, plus the component of this lateral deflection. Hence:

$$u = y_b\sin\phi + \frac{w\sin\phi L^4}{120EI_y}\cos\phi \qquad ....(3)$$

where y_b is the height of the centroid above the soffit.

This lateral deflection will cause a corresponding moment about the bearings of

$$M = wLu \qquad ....(4)$$

On the assumption that the beam is supported by a bearing at each end, each of which has a rotational stiffness of K:

$$M = 2\phi K \qquad ....(5)$$

Thus, the beam can topple sideways by rotation of the bearing if

$$wL\left[y_b\sin\phi + \frac{w\sin\phi L^4}{120EI_y}\cos\phi\right] = 2\phi K \qquad ....(6)$$

Without any loss of generality, ϕ can be taken to be small through the initial buckling increment, so $\sin\phi = \phi$ and $\cos\phi = 1$. Thus,

$$(wL)_{cr}^2\frac{L^3}{120EI_y} + (wL)_{cr}y_b - 2K = 0 \qquad ....(7)$$

This is a quadratic in $(wL)_{cr}$, from which the critical load can be obtained. This critical load is the self-weight that would cause buckling of a perfect beam. Thus, the ratio w_{cr}/w is the safety margin for the beam against toppling caused by rotation of the supports.

Two special cases are worthy of note. If the beam is rigid, the quadratic term is zero, so

$$(wL)_{cr} = \frac{2K}{y_b} \qquad ....(8)$$

while, if the beam is very flexible, the linear term can be ignored, so

$$(wL)_{cr} = \sqrt{\left(\frac{240EI_yK}{L^3}\right)} \qquad ....(9)$$

In practice, eqn (7) will be used to check the stiffness of the bearing as described below.

It should also be noted that any lateral misplacement of the beam upon a bearing could lead to toppling, as shown by Mast[6].

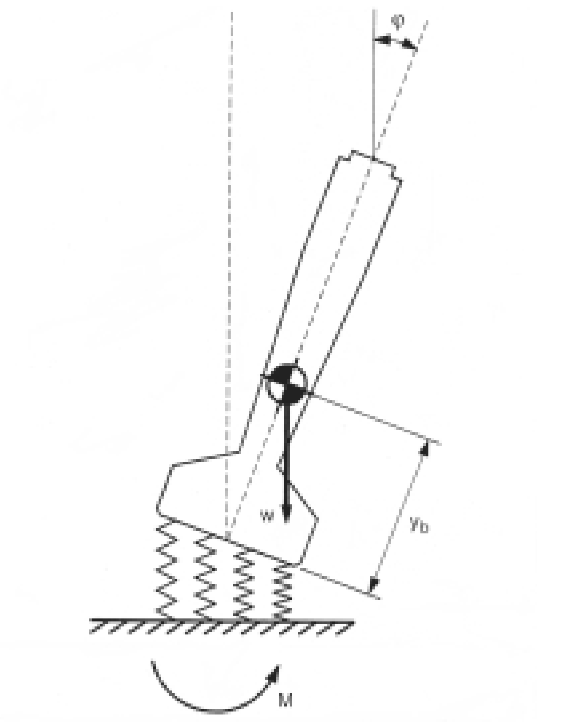


Fig 1. Rotation of beam on bearings

Behaviour of beams with imperfections

The analysis so far has considered perfect beams, but real beams have imperfections. These initial imperfections can be magnified by second-order effects, and it is the curvatures associated with these deflections that can lead to cracking of the concrete in tension. This, in turn, can lead to a greatly reduced minor-axis stiffness, which might then lead to sudden collapse. Some knowledge of the way these imperfections can grow is thus of major importance; this can be provided by making use of the Southwell construction.

Southwell plot

Southwell[7] gave a method by which the buckling load and the value of the initial imperfection of a strut could be determined from measurements of the way the lateral deflections vary at loads below the critical load. The method was intended as an experimental tool to determine the buckling load of a perfect structure when imperfections are present, but the method can be reversed to predict the deflections of an imperfect structure. If the buckling load is known, and the magnitude of the initial imperfection can be assumed (or limited in a specification), the lateral deflection can be found from

$$v = \frac{v_o}{1 - \dfrac{w}{w_{cr}}} \qquad ....(10)$$

The rotation of the beam ϕ can then be found by assuming that the additional deflection, $v - v_0$, must be induced by the lateral component of the beam's self-weight, so:

$$v - v_o = \frac{5wL^4\sin\phi}{384EI_y} \qquad ....(11)$$

The minor-axis curvature of the beam at midspan can then be found simply by considering the component of the midspan bending moment acting about the minor axis.

$$\kappa_{ms} = \frac{w\sin\phi L^2}{8EI_y} \qquad ....(12)$$

Eqns (10), (11) and (12) can be combined to give a direct relationship between the load and the curvature

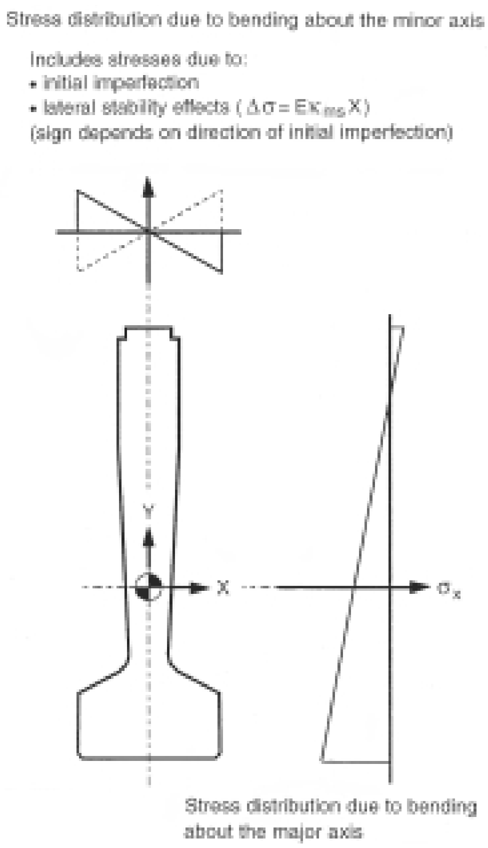


Fig 2. Summation of stresses due to bending about both axes

TABLE 1: Rotational stiffness

	Variable	Units	M-10 (29m long)	SY-6 (40m long)
Required stiffness of bearings	K	kNm/rad	2644	27957
Additional stress	Δσ	MPa	0.50	0.43

$$\kappa_{ms} = \frac{48 v_0}{5L^2}\left(\frac{1}{\frac{w_{cr}}{w} - 1}\right) \qquad ....(13)$$

The value of v_0 used in this equation should be the sum of the initial lack of straightness of the beam as manufactured, plus the misplacement of the beam on the bearings. Both values will have to be assumed at the time of design, but reasonable values can be taken for both.

Eqn (13) shows the importance of the term w/w_{cr}. If this term approaches unity, the denominator tends to zero and the curvature becomes very large.

It is expected that fairly conservative factors of safety would be applied to the ratio w/w_{cr} since, at the design stage, there is no knowledge of the imperfections that will occur on site.

The curvature can be used to determine the stress distribution across the beam; at a distance X from the beam's minor axis the change in the concrete stress $\Delta\sigma$ can be found from:

$$\Delta\sigma = E\kappa_{ms}X \qquad ....(14)$$

This stress must be superposed on the major-axis stress distribution, allowing the stress at two critical points to be found, as shown in Fig 2. These critical points will normally be at the corners of the section and will give the largest tensile and compressive stresses.

In practice, additional compressive stresses are unlikely to be a problem, but additional tensile stresses must be compared with the precompression to ensure that cracking does not occur. Cracking would lead to a reduction in minor-axis stiffness EI_y, which, in turn, would lead to a reduction in the buckling load w_{cr} from eqn (7) and an increase in the curvature from eqn (13).

Examples

The M10 is the largest beam in the M series[8]. It is designed to have a maximum length of 29.5m.

The SY series of beams is the largest standard series of precast beams currently manufactured in the UK[9]. They are narrower, deeper, longer and heavier than the M series beams and, on each count, can be expected to be more susceptible to stability problems. The maximum recommended length for these beams is 40m.

The initial imperfections are taken to be span/1000 for the lack of straightness of the beams themselves and 10mm for the misplacement of the beam on the bearing. These are typical of the values that can be obtained by good quality control and careful workmanship; higher values might be expected with less care.

Table 1 shows the rotational stiffness required from bearings needed to provide a factor of 10 reserve of strength against rotation of the beam on its supports. Such a high factor of safety might seem excessive at first sight, but it is justified by the additional stresses that are caused by the additional midspan curvature given by eqn (13); such stresses are not normally taken into account at the design stage, and it is relatively cheap to avoid them by choice of a suitable bearing. The rotational stiffness required varies from 2650kNm/rad for the M10 beam to 28 000kNm/rad for the 40m-long SY6 beam. At these stiffnesses, the additional stresses induced by the imperfections is negligible, but, if the reserve factor against toppling is reduced, these additional stresses can become significant.

Fig 3 shows the contrasting effects of choosing different values for w/w_{cr} for a SY6 beam at a span of 40m. At low values, a stiff bearing is required but the stresses induced by growth of the imperfections are low and can probably be ignored. However, if w_{cr} is chosen to be only slightly higher than the self-weight of the beam, a lower stiffness bearing is required, but the loss of precompression can become quite high. The choice of appropriate value is up to the designer.

Rotational stiffness of laminated rubber bearings

The rotational stiffness of bearings used here has dimensions of moment/rotation, and not force/rotation, which is the value quoted in many bearing catalogues. This latter value relates to the rotation of the bearing caused by a transverse force. In general, values of K are not quoted by manufacturers. Information is available in the specialist literature[10,11], but this is sometimes difficult to obtain. For convenience, the information below is taken from Gent & Meinicke[11].

Consider a section through a thin sheet of rubber, of unstressed thickness t, constrained between two rigid plates (Fig 4). A moment is applied to the top surface, as shown, and causes a rotation ϕ. The rotational stiffness for this layer is

Fig 3. Effect of changing factor of safety on required bearing stiffness and the induced minor-axis stresses

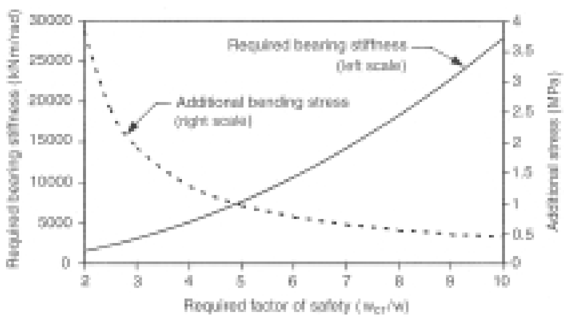


Fig 4. Rotation of restrained layer of rubber

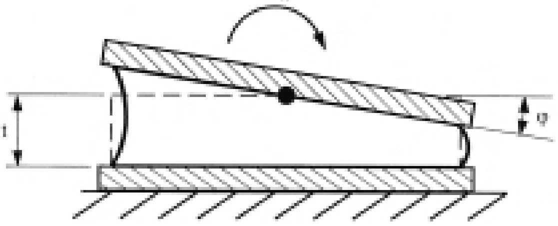


TABLE 2: f_{b1} and f_{b2} formulae

Cross-section	f_{b1}	f_{b2}
Circle, radius r	1	$\frac{r^2}{6t^2}$
Ellipse, semi-axes a & b, bent about the $2b$ axis	$\frac{4}{3} - \frac{2\left(\frac{ab}{2} + t^2\right)}{3\left(\frac{a^2}{4} + b^2 + 2t^2\right)}$	$\frac{2a^2b^2}{3t^2(a^2 + 3b^2)}$
Square, side $2a$	1	$\frac{0.0464(2a)^2}{t^2}$
Rectangle, sides $2a$ & $2b$, bent about axis parallel to $2b$ side	$\frac{4}{3} - \frac{2\left(\frac{ab}{2} + t^2\right)}{3\left(\frac{a^2}{4} + b^2 + 2t^2\right)}$	$\frac{6(2a)^2}{\pi^4 t^2} \sum_{j=1}^{\infty} \frac{1}{j^4} \left[1 - \frac{\tanh\left(\frac{j\pi b}{a}\right)}{\left(\frac{j\pi b}{a}\right)}\right]$

given by M/ϕ.

$$K = \frac{M}{\phi} = \frac{f_b A k^2 E_R}{t} \qquad ....(15)$$

where

A is the plan area of the rubber sheet
k is the radius of gyration of the plate
E_R is the Young's modulus of the rubber as measured in an unconfined sample

The effect of the constraint caused by the steel layers (which are effectively rigid by comparison with the rubber) is allowed for by the factor f_b, which is the ratio of the apparent value of Young's modulus of the constrained sheet to E_R. It is the sum of two components

$$f_b = f_{b1} + f_{b2} \qquad ....(16)$$

which depend on the shape of the bearing. Formulae for the values of f_{b1} and f_{b2} are given in Table 2.

The stiffness of a bearing with n layers is simply $1/n$ times the stiffness of a single layer. Applying this method to bearings taken from a manufacturer's catalogue, and taking E_R for rubber as 2.4N/mm², gives values for K ranging from 26kNm/rad for a 100mm square bearing with three layers, up to 853 000kNm/rad for a 900mm square bearing with only one layer. It is thus clear that the required values for typical beams, as given in Table 1, are in the same range as the values of stiffness quoted here. Thus, the bearing rotational stiffness should be checked or specified at the design stage.

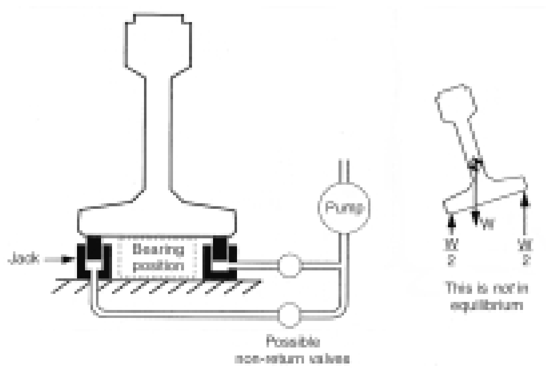
Jacking systems

Beams are often placed on temporary supports by crane and then lifted onto their permanent bearings by means of jacks. One common arrangement consists of placing two jacks under the outer edges of the bottom flange which can then bridge the permanent bearing. Both jacks are then connected to a single handpump, as shown in Fig 5. It must be emphasised that this arrangement does not provide any rotational restraint, and indeed, if the beam rotates, the system can cause toppling, since the hydraulic system ensures that both jacks exert the same force, whereas, if the beam's centroid is displaced at all, equilibrium would make the forces different.

In effect, such a system acts as a pin with negative stiffness, and it can be stable only if the whole of the rotational restraint is provided by the support at the far end of the beam. This then requires a rotational stiffness much higher than that calculated from eqn (7).

Placing non-return valves into the hydraulic system in Fig 5 does not solve the problem. Once the beam starts to tilt, the jack on the lower side effectively becomes fixed, and all the flow is directed into the jack on the higher side, which thus extends, increasing the tilt. Having separate pumps for the two sides solves the problem, but the system then becomes much harder to control.

Fig 5. Lack of equilibrium in jacking system



Temporary supports

The principles outlined earlier also apply to temporary support arrangements, where conventional bearings are not normally used, and it should also be noted that several of the recent beam failures[4,5] have related to beams toppling off supports during installation.

Conclusions

Long, precast concrete beams can be susceptible to lateral instability during erection, if insufficient restraint is provided against lateral rotation. Imperfections, both in the beam itself and in its placement on bearings, can lead to minor-axis curvatures in the beam and in a reduction of the precompression.

The required stiffness of the permanent bearings can be calculated easily, and should be specified at the time of design. However, because of the highly complex interaction of beam imperfection, bearing misalignment, and buckling mode, it is probably desirable that a large reserve of rotational stiffness is provided by the bearing. Care should also be taken to place the beam centrally upon the bearing.

The problems of lack of rotational restraint apply equally to temporary support conditions. If slender beams are to be placed on temporary supports, they should be propped against lateral rotation until the permanent bracing arrangements are in place. Care should be exercised that the temporary supports are level. The use of waste timber as temporary packing should be forbidden.

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