UNIVERSITY OF CAMBRIDGE

ECONOMICS TRIPOS PART I

Friday 15 June 2012 9:00-12:00

Paper 3

QUANTITATIVE METHODS IN ECONOMICS

There will be a 15 minute reading time prior to the beginning of the examination.

This paper is divided into four Sections:
Sections A and B are Mathematics; Sections C and D are Statistics.

You should do the appropriate number of questions from each Section.
The number of questions to be attempted is at the beginning of each Section.
Answer all parts of the question.

Answers from the Mathematics and the Statistics Sections must be written in separate booklets with the letter of the Section written on each cover sheet.

This written exam carries 80% of the marks for Paper 3.

Section A carries 24% of the marks
Section B carries 16% of the marks
Section C carries 24% of the marks
Section D carries 16% of the marks

Write legibly.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
20 Page booklet x 2	List of statistical formulae
Metric graph paper	New Cambridge Elementary Statistical Tables
Rough work pads	Approved calculators allowed
Tags	

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A - Answer ALL FOUR questions from this Section.

1 A common utility function is

$$U(w) = \frac{w^{1-\alpha} - 1}{1 - \alpha}$$

where $w > 0$ is wealth and α is known as the coefficient of risk aversion.

(a) For what values of α is the function concave.

(b) This utility function is not defined at $\alpha = 1$. If we want the marginal utility of wealth, that is $MU(w) = U'(w)$, to be given by the same function at $\alpha = 1$ as for all $\alpha \neq 1$, then how should we define the utility function $U(w)$ for $\alpha = 1$?

(c) Now, assume that an individual with the utility function above lives forever, receives $w = 27$ in each period, and discounts the future at a rate $\beta = 0.5$. Hence, his life-time utility is given by

$$U(w) = \sum_{t=1}^{\infty} \beta^{t-1} \left[\frac{w^{1-\alpha} - 1}{1 - \alpha} \right]$$

What is the value of his life-time utility if $\alpha = \frac{2}{3}$?

2 Imagine that a person can spend his time on the three activities x, y and z, and that his utility is given by

$$U(x, y, z) = \sqrt{x} + \frac{1}{4}y + \ln(1 + z) \qquad \text{if } x, y, z \geq 0$$

(a) Assume that the time available to the person is normalised to one, and that only the activities x and y are available to him. That is, $x + y \leq 1$, $x, y \geq 0$ and $z = 0$.

Set up the utility maximisation problem, and solve it for the values of x, y and U *without* using the Lagrangian method.

(b) Now, assume that also the third activity z available. Further, assume that time is measured in hours and that the time available to the person is exactly 24 hours. That is, $x + y + z = 24$, and $x, y, z \geq 0$.

Set up the utility maximisation problem, and solve it for the values of x, y, z and U using the Lagrangian method.

3 Let A be a 2×2 square matrix given by

$$A = \begin{bmatrix} 5 & 2 \\ -\alpha & 3 \end{bmatrix}$$

(a) Calculate the determinant of A, and invert the matrix. For what values, if any, of α is A positive definite?

(b) The function

$$5x^2 + 3y^2 + 4xy$$

has an extreme point. Is this extreme point a minimum? If possible, use what you learnt in (a) to help you answering the question.

4 (a) Let $f(x) = \ln(x)$ for all $x > 0$. Find the second order Taylor expansion for f around the point $x_0 = 1$.

(b) Let $x_t = bx_{t-1} + a$ for all $t = 1, 2, \ldots$, where a is a constant and $0 < b < 1$. Does the sequence converge? Explain.

(c) Find the function $y(t)$ that satisfies $y(0) = 0$, $y'(0) = -1$ and $y''(t) = 2$ for all $t \in [0, 1]$.

(TURN OVER)

SECTION B - Answer ONE question from this Section.

5 Assume that an individual is going to invest his wealth in a portfolio consisting of two risky assets, and that his utility is given by

$$U = \delta E(R_1) + (1-\delta)\, E(R_2) - \alpha \left(\delta^2 \sigma_1^2 + (1-\delta)^2 \sigma_2^2 + 2\delta (1-\delta) \sigma_{12}\right)$$

where, $\alpha > 0$ is a measure risk tolerance. $E(R_1)$ and $E(R_2)$ are the expected returns of assets 1 & 2, while σ_1^2 and σ_2^2 are their variances and σ_{12} their covariance. Wealth is normalised to one, that is the portfolio consists of a fraction δ of asset 1 and a fraction $(1-\delta)$ of asset 2, where $\delta \in [0,1]$.

(a) If $E(R_1) \neq E(R_2)$ and $\sigma_1^2 = \sigma_2^2 = \sigma_{12} = 1$, how would the individual choose to invest his assets if he wanted to maximise his utility? Explain.

(b) More generally, the optimal weight of asset 1 in the portfolio is given by

$$\delta = \frac{E(R_1) - E(R_2) + 2\alpha\left(\sigma_2^2 - \sigma_{12}\right)}{2\alpha\left(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}\right)}$$

if the expression results in a $\delta \in [0,1]$. Solve the investment problem and demonstrate that this is indeed true if we make the appropriate assumptions. Discuss what would happen if $\delta < 0$ and if $\delta > 1$.

(c) Assume that $E(R_1) = 15$, $E(R_2) = 10$, $\sigma_1^2 = 50$, $\sigma_2^2 = 25$, $\sigma_{12} = 0.5$ and $\alpha = 2$. Calculate the optimal δ^*, and perform comparative statics with respect to σ_2^2, that is the variance of asset 2. What is going on?

6 In an economy, there are two firms, one domestic firm D and one foreign firm F. Both firms have the same cost function

$$c(q_i) = 3q_i$$

and the same demand function, where q_i is the quantity produced by firm $i = D, F$.

$$D(p) = 100 - 8p$$

and $D(p) = Q = q_D + q_F$ and p is the price. Finally, profits are given by

$$\pi_i(q_i, p) = pq_i - c(q_i)$$

(a) Assume that the two firms are competing by simultaneously choosing the quantity that maximises their profits. Solve for Q and $\pi_i(q_i, p)$.

(b) Now, assume instead that there are n identical firms in the economy, that is $i = 1, 2, ..., n$. Set up the maximisation problem of a generic firm i.

(c) What happens to prices, p, and profits, π_i, as the number of identical firms in the economy goes to infinity? Demonstrate by solving the above problem with n firms and taking the limits as n goes to infinity.

SECTION C - Answer ALL FOUR questions from this Section.

7 Kris is going to a party. There is a 30% chance that Kim will go to the same party. If Kim does not go, there is a 70% chance that Kris will enjoy himself. If Kim goes, there is only a 10% chance that Kris will enjoy himself.

(a) What is the probability that Kris will enjoy the party?

(b) Suppose you know that Kris did enjoy the party. What is the probability that Kim was not present?

(c) Suppose you know that Kris did not enjoy the party. What is the probability that Kim was present?

8 A probability density function of a continuous random variable, X, is given by

$$f_X(x) = \begin{cases} c_1 x^2 + c_2 & \text{for } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

for some unknown values of c_1 and c_2. You are told that the expected value of X is $1/4$.

(a) Determine the values of c_1 and c_2.

(b) Derive the cumulative distribution function of X.

(c) What is $\Pr(X > 1/2)$?

9 Let $Y_1, \ldots, Y_{25}$ be a random sample from $N(\mu, 6.25)$.

(a) Construct a test procedure of size 0.05 and test the null hypothesis that $\mu = 10$ against the alternative $\mu < 10$.

(b) Evaluate the power of your test in part (a) when μ takes the following values $9, 10, 11$. Briefly comment on your answers.

(c) Without any further calculation, indicate how the power of the test you have calculated in part (b) may change if the sample size is larger than 25.

10 Consider the following estimated regression of output (Y) on fixed capital (K) from 400 factories:

$$\widehat{\ln Y} = \underset{(2.61)}{0.79} + \underset{(0.015)}{1.015} \ln K,$$

( ) denotes the standard error of the parameter estimate immediately above it.

(a) Interpret the estimate of the slope parameter of the estimated regression, and comment on the sign and its magnitude.

(b) Test at 5% significance level whether there is a constant return to scale in the production technology. Clearly state all assumptions you are making.

(c) Provide a 95% confidence interval for the slope parameter and comment on your answer.

(TURN OVER)

SECTION D - Answer ONE question from this Section.

11 X is distributed as a uniform random variable on $[0, \theta]$, for some $\theta > 0$. Let $X_1, \ldots, X_n$ be a random sample of X.

(a) Derive $E(X)$ and $Var(X)$ in terms of θ.

(b) Is the sample mean an unbiased estimator for θ? If it is not unbiased, using the sample mean or otherwise, construct an unbiased estimator for θ.

(c) Provide a consistent estimator for θ and show that it is consistent. (Hint: Compute the mean squared error)

(d) Suppose that $n = 100$. Researcher A is provided with only the first fifty elements of the sample, namely $X_1, \ldots, X_{50}$, whilst Researcher B is provided with the full sample. Both Researchers A and B are instructed to employ the unbiased estimator you proposed in part (b). Discuss in detail how the property of their estimators differ. Which of the two estimators would you prefer? Explain your reasoning.

12 Consider a linear consumption (C_i) function in income (I_i)

$$C_i = \beta_0 + \beta_1 I_i + \varepsilon_i$$

Using observations from 500 families on annual consumption and income (measured in US dollars), (β_0, β_1) are estimated using ordinary least squares with the following fitted regression

$$\begin{aligned} C_i &= \underset{(142.7)}{-131.2} + \underset{(0.017)}{0.84} I_i, \text{ and } R^2 = 0.691, \end{aligned}$$

( ) denotes the standard error of the parameter estimate immediately above it, and R^2 is the coefficient of determination.

(a) Interpret the estimate of β_0 and comment on its sign and magnitude.

(b) Interpret the estimate of β_1 and comment on its sign and magnitude.

(c) Under what assumptions are the least squares estimators unbiased?

(d) What is the predicted level of consumption when a family income is \$30,000?

The *average expected propensity to consume* (APC) for a level of income I is $\beta_0/I + \beta_1$.

(e) Sketch the estimated APC with I on the x-axis.

(f) Test whether APC depends on the level of income. Clearly state all assumptions you are making.

(g) Suppose the data also contain information whether the families are either in a northern or southern states, and you suspect there might be a difference in their consumption behaviour. Suggest what regression(s) you may want to run and the hypothesis test(s) you may want to conduct. Explain your answers.

END OF PAPER